Eur. Phys. J. B **51**, 373–376 (2006)
DOI: 10.1140/epjb/e2006-00229-3

THE EUROPEAN
PHYSICAL JOURNAL B

Correlated noises in an absorptive optical bistable model

B.-Q. Ai[1,a], H. Zheng[1], and L.-G. Liu[2]

[1] School of Physics and Telecommunication Engineering, South China Normal University, 510006 GuangZhou, P.R. China
[2] Faculty of Information Science, Macao University of Science and Technology, Macao, P.R. China

Received 30 January 2006 / Received in final form 7 March 2006
Published online 14 June 2006 –

Abstract. We study the steady state properties of an absorptive optical bistable model in the presence of correlated noises. Based on the corresponding Fokker-Planck equation the steady state solution of the probability distribution and the average value of the transmitted light have been investigated. We have found that fluctuations of the input light amplitude improve the transmitted light and an optimized value exists for the fluctuations of the population difference at which the transmitted light takes its maximum value. The correlation between the two noises reduce the transmitted light and the noises in the model can induce a phase transition.

PACS. 05.40.-a Nonlinear dynamics and chaos – 02.50.Ey Stochastic processes

1 Introduction

Recently, nonlinear stochastic systems with noise terms have been the subject of extensive investigations. The concept of noise-induced transition has many applications in the fields of physics, chemistry and biology [1,2]. In these systems the noise usually affects the dynamics through a system variable, i.e., the noise is both multiplicative and additive [3]. The focal theme of these investigations is to study the steady state properties of systems in which fluctuations, generally applied from outside, are considered independent of the system's characteristic dissipation. Since the two types of fluctuations have a common origin, they are correlated in the relevant timescale of the problem [4]. On the level of a Langevin-type description of a dynamical system, the presence of correlation between noise can change the dynamics of the system [5,6]. Correlated noise processes have found applications in a broad range of studies such as steady state properties of a single mode laser [7], bistable kinetics [8], directed motion in spatially symmetric periodic potentials [9], stochastic resonance in linear systems [10], and steady state entropy production [11]. In this paper we study an absorptive optical bistable model in the presence of the correlated noises, and show how noise can dynamically affect the optical system.

[a] e-mail: aibq@scnu.edu.cn

2 Stationary probability distribution in an absorptive optical bistable model

Consider a ring interferometer with a passive medium placed in it. Light is coupled into the interferometer through a semipermeable mirror and, likewise, light is transmitted at another mirror. Measuring the intensity of the transmitted wave against the intensity of the incident wave, one finds an S-shaped curve; e.g., for some values of the intensity of the incident beam the intensity of the transmitted wave can have a small and a large intensity. There are different mechanisms that can be responsible for this behavior. One of them is due to nonlinear absorption in the passive medium. A model for purely absorptive optical bistability in a cavity was introduced by Bonifacio and Lugiato [12]. For the scaled dimensionless amplitude y of the input light and the scaled dimensionless amplitude of the transmitted light x, the equation of motion — in the adiabatic limit of atomic variables can be derived [12]

$$\frac{dx}{dt} = y - x - \frac{2cx}{1+x^2}, \tag{1}$$

where c is proportional to the population difference in the two relevant atomic levels and is controlled by varying the density, the length and reflectivity.

Figure 1 shows the stationary transmitted light amplitude x as a function of the input light amplitude y for different values of c. When $c \leq 4$, equation (1) has only one solution for all values of y. When $c > 4$, equation (1) may

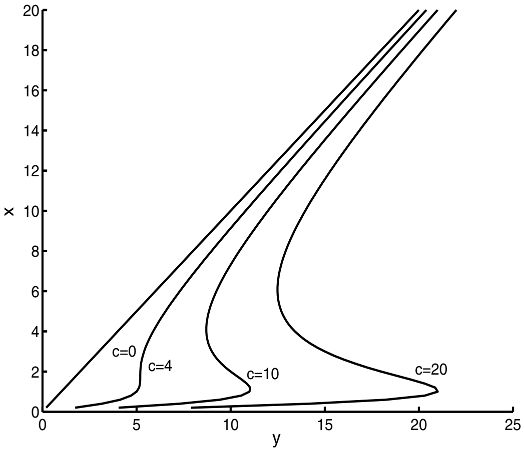

Fig. 1. The stationary transmitted light amplitude x vs. the input light amplitude y for different values of $c = 0, 2, 4, 10, 20$.

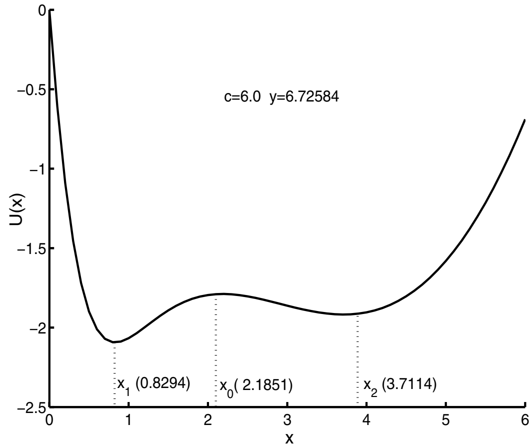

Fig. 2. The potential $U(x)$ vs. x for $c = 6$ and $y = 6.72584$.

have three solutions for an appropriate value of y. When $c = 0$, x is equal to y. For a large enough population difference c, the stationary transmitted light amplitude is an S-shaped function of the amplitude of the input light, thus indicating bistability.

In this paper, we let $c = 6$ and $y = 6.72584$. The associated potential:

$$U(x) = \frac{1}{2}x^2 - yx + c\ln(1 + x^2), \tag{2}$$

is shown in Figure 2. From Figure 2, we can see that the system is bistable and has two stable states $x_1 = 0.8294$, $x_2 = 3.7114$ and an unstable state $x_0 = 2.1851$.

Now, if some environmental external disturbances make both the input light amplitude and the density or population difference fluctuate, they are likely to affect y and c in the form of an additive noise and a multiplicative noise that are connected through a correlation parameter λ. As a result we have

$$\frac{dx}{d\tau} = y - x - \frac{2cx}{1 + x^2} - \frac{x}{1 + x^2}\Gamma(t) + \xi(t). \tag{3}$$

Where $\Gamma(t)$ and $\xi(t)$are Gaussian white noises with the following properties [14]:

$$\langle \Gamma(t) \rangle = \langle \xi(t) \rangle = 0, \tag{4}$$

$$\langle \Gamma(t)\Gamma(s) \rangle = 2D\delta(t - s), \tag{5}$$

$$\langle \xi(t)\xi(s) \rangle = 2\sigma\delta(t - s), \tag{6}$$

$$\langle \xi(t)\Gamma(s) \rangle = \langle \Gamma(t)\xi(s) \rangle = 2\lambda\sqrt{D\sigma}\delta(t - s). \tag{7}$$

where D and σ are the strength of noise $\Gamma(t)$ and $\xi(t)$, respectively, and λ denotes the degree of correlation between $\Gamma(t)$ and $\xi(t)$ with $0 \le \lambda \le 1$.

We can derive the corresponding Fokker-Planck equation for the evolution of the probability distribution based on equations (3)–(7). Adopting Stratonovich's interpretation, this equation satisfies [13,14]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial A(x)P(x,t)}{\partial x} + \frac{\partial^2 B(x)P(x,t)}{\partial x^2}, \tag{8}$$

where

$$A(x) = h(x) + Dg_1(x)g_1^{'}(x) + \lambda\sqrt{D\sigma}g_1(x)g_2^{'}(x) + \lambda\sqrt{D\sigma}g_1^{'}(x)g_2(x) + \sigma g_2(x)g_2^{'}(x), \tag{9}$$

$$B(x) = Dg_1^2(x) + 2\lambda\sqrt{D\sigma}g_1(x)g_2(x) + \sigma g_2^2(x), \tag{10}$$

here $h(x) = y - x - \frac{2cx}{1+x^2}$, $g_1(x) = -\frac{x}{1+x^2}$, $g_2(x) = 1$. The steady probability distribution of the Fokker-Planck equation is given by [14]

$$P_{st}(x) = \frac{N_0}{B(x)}\exp\left[\int^x \frac{A(x^{'})}{B(x^{'})}dx^{'}\right]. \tag{11}$$

where N_0 is the normalization constant. The average amplitude of transmitted light $\langle x \rangle$ can be determined by

$$\langle x \rangle = \frac{\int_0^\infty x\ p_{st}(x)dx}{\int_0^\infty p_{st}(x)dx}. \tag{12}$$

3 Results and discussion

Figure 3a shows the effect of the noise intensity σ of the input light on the steady state probability distribution. When $D = 0$ and $\lambda = 0$, only one noise — the fluctuation of the input light–acts on the system. When σ is small (0.05), the effect of the potential dominates and $U(x_1) < U(x_2)$, so the peak at x_1 is very high. When σ increases, the peak at x_1 decreases and the peak at x_2 increases. The peaks flatten out and almost vanish for a large enough value of σ, indicating that the additive noise is a diffusive term. The positions of the peaks are weakly affected by σ. Figure 3b shows the average transmitted light amplitude as a function of σ. The average value of x increases with the fluctuation σ of the input light. For a large enough value of σ, the fluctuation dominates, and saturates to a certain value. Similar asymmetry effects were reported in Borromeo and Marchesoni' work [15].

Figure 4a shows the steady state probability density $P_{st}(x)$ as a function of the transmitted light amplitude x

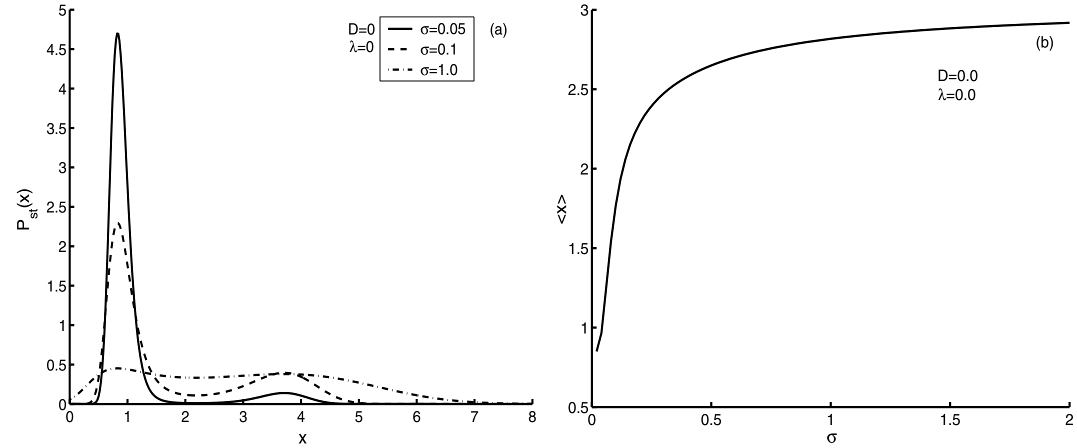

Fig. 3. (a) Steady state probability density $P_{st}(x)$ vs. the transmitted light amplitude x for different values of $\sigma = 0.05, 0.1, 1.0$ at $D = 0$ and $\lambda = 0$. (b) The average amplitude of the transmitted light $\langle x \rangle$ vs σ at $D = 0$ and $\lambda = 0$.

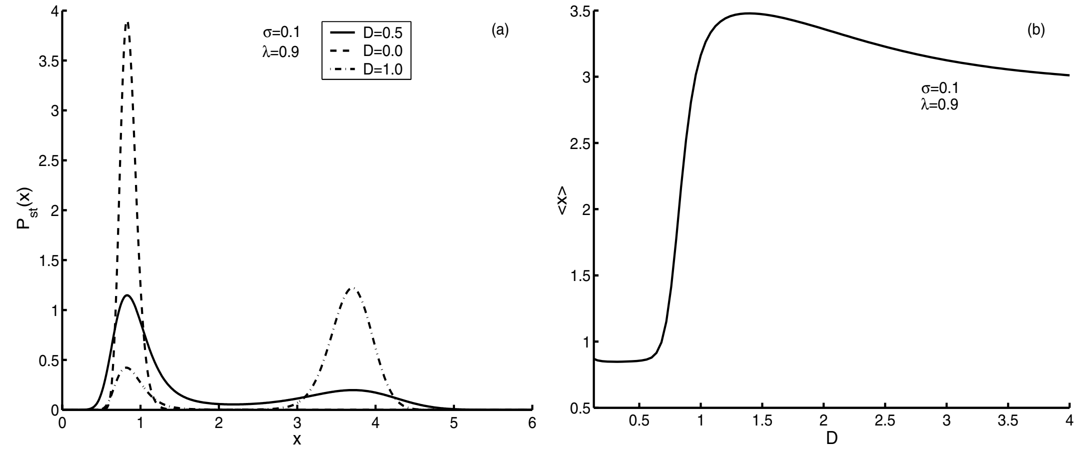

Fig. 4. (a) Steady state probability density $P_{st}(x)$ vs. the transmitted light amplitude x for different values of $D = 0.0, 0.5, 1.0$ at $\sigma = 0.1$ and $\lambda = 0.9$. (b) The average amplitude of the transmitted light $\langle x \rangle$ vs. D at $\sigma = 0.1$ and $\lambda = 0.9$.

for different values of D. When $D = 0$, namely no fluctuation of population difference c, there is only one peak at x_1. When D increases, the height of the peak at x_1 decreases and a new peak appears at x_2. The system evolves from a single state system to a bistable system. The multiplicative noise is a drift term. The average value of x as a function of D is shown in Figure 4b. When D is very small, the system is in one state, so $\langle x \rangle$ has no change with D. When D increases, the system becomes bistable, so $\langle x \rangle$ increases drastically with increasing D. For a large value of D, the system effects disappear, $\langle x \rangle$ decreases to a certain value. There is an optimum value of D at which the average value of x takes its maximum value.

In Figure 5a, we show the effect of correlation parameter λ on the steady state probability distribution. When $\lambda = 0$ there are two approximately equal-height peaks at x_1 and x_2. As the value of λ increases, the peak at x_1 increases, while the peak at x_1 decreases. The steady state probability distribution is thus bistable for $\lambda = 0$ and becomes monostable for larger values of λ. The average value of x as a function of λ is plotted in Figure 5b. It decreases as λ increases. Since x denotes the amplitude of transmitted light, it is clear that the correlation between the noise reduces the amplitude of transmitted light.

4 Concluding remarks

In this paper, we study the steady state properties of an absorptive optical bistable model in the presence of two correlated noise. The effects of the fluctuations in the population difference (multiplicative noise) and in the input light amplitude (additive noise) are investigated. Fluctuation of the input light can improve the transmitted light. The fluctuation of the population difference has an optimum value at which the transmitted light amplitude takes its maximum value. Correlation between the noises reduces the transmitted light. On the other hand, the noise can dynamically affect the system. The noise from population difference fluctuations makes the system switch from

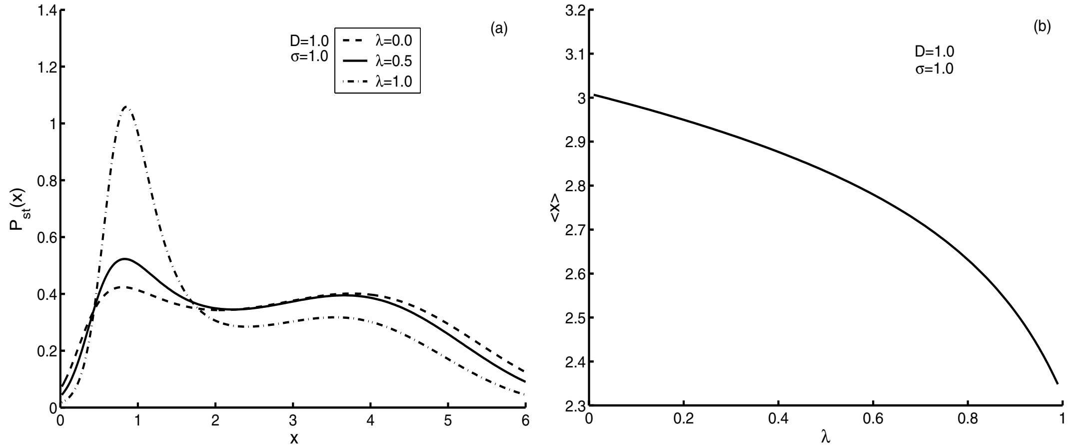

Fig. 5. (a) Steady state probability density $P_{st}(x)$ vs. the transmitted light amplitude x for different values of $\lambda = 0.0, 0.5, 1.0$ at $D = 1.0$ and $\sigma = 1.0$. (b) The average amplitude of the transmitted light $\langle x \rangle$ vs. λ at $D = 1.0$ and $\sigma = 1.0$.

monostable to bistable behavior, while the correlation between the two noises induces a transition in the system from bistable to monostable behavior. In the absorptive optical bistable model, the noise from the fluctuation of the parameters can induce a phase transition.

References

1. A. Fulinski, T. Telejko, Phys. Lett. A **152**, 11 (1991)
2. B.Q. Ai, X.J. Wang, G.T. Liu, L.G. Liu, Phys. Rev. E **68**, 061105 (2003); B.Q. Ai, X.J. Wang, G.T. Liu, L.G. Liu, Phys. Rev. E **67**, 022903 (2003); B.Q. Ai, G.T. Liu, H.Z. Xie, L.G. Liu, Chaos **14**(4), 957 (2004)
3. F. Marchesoni, P. Grigolini, Physica A **121**, 269 (1983); R. Bartussek, P. Reimann, P. Hanggi, Phys. Rev. Lett. **76**, 1166 (1996)
4. W. Hersthemke, R. Lefever, *Noise-induced Transitions* (Springer-Verlag, Berlin, 1984)
5. S.K. Banik, *Correlated Noise Induced control prey extinction* (Physics/0110088), (2001)
6. L. Cao, D.J. Wu, Phys. Lett. A **185**, 59 (1994)
7. S. Zhu, Phys. Rev. A **47**, 2405 (1993)
8. Y. Jia, J.R. Li, Phys. Rev. E **53**, 5786 (1996)
9. J.H. Li, Z.Q. Huang, Phys. Rev. E **53**, 3315 (1996)
10. V. Berdichevsky, M. Gitterman, Phys. Rev. E **60**, 1494 (1999)
11. B.C. Bag, S.K. Banik, D.S. Ray, Phys. Rev. E **64**, 026110 (2001)
12. R. Bonifacio, L.A. Lugiato, Phys. Rev. A **18**, 1192 (1978); L. Gammaitoni, P. Hanggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. **71**, 223 (1998)
13. D.J. Wu, L. Cao, S.Z. Ke, Phys. Rev. E **50**, 2496 (1994)
14. H. Risken, *The Fokker-Planck equation* (Springer, Berlin 1984)
15. M. Borromeo, F. Marchesoni, Europhys. Lett. **68**, 783 (2004); M. Borromeo, F. Marchesoni, Phys. Rev. E **71**, 031105 (2005); M. Borromeo, F. Marchesoni, Europhys. Lett. **72**, 362 (2005)